Chapter 5: Radical Expressions & Equations

1. Express $3xy\sqrt[3]{2x}$ as an entire radical.

$= \sqrt[3]{2x \cdot 3^3 \cdot x^3 \cdot y^3}$

$= \sqrt[3]{54x^4y^3}$

2. Express $\sqrt{48a^3b^2c^5}$ as a simplified mixed radical.

$= \sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot a \cdot a \cdot a \cdot b \cdot b \cdot c \cdot c \cdot c \cdot c \cdot c}$

$= 4abc^2\sqrt{3ac}$

3. Order the set of numbers from least to greatest.

$$3\sqrt{6}, \sqrt{36}, 2\sqrt{3}, \sqrt{18}, 2\sqrt{9}, \sqrt[3]{8}$$

$= \sqrt{54}, \sqrt{36}, \sqrt{12}, \sqrt{18}, \sqrt{36}, \sqrt[3]{8}$

$\sqrt[3]{8} < \sqrt{12} < \sqrt{18} < \sqrt{36} \leq \sqrt{36} < \sqrt{54}$

4. Simplify each expression. Identify any restrictions on the values for the variables.

a) $4\sqrt{2a} + 5\sqrt{2a}$

$= 9\sqrt{2a}$

$a \geq 0$

b) $10\sqrt{20x^2} - 3x\sqrt{45}$

$= 10\sqrt{2 \cdot 2 \cdot 5 \cdot x \cdot x} - 3x\sqrt{5 \cdot 3 \cdot 3}$

$= 20x\sqrt{5} - 9x\sqrt{5}$

$= 11x\sqrt{5}$

no restrictions

5. Simplify. Identify any restrictions on the values of the variable in part c).

a) $2\sqrt[3]{4}\left(-4\sqrt[3]{6}\right)$

$= -8\sqrt[3]{24}$

$= -8\sqrt[3]{2 \cdot 2 \cdot 2 \cdot 3}$

$= \boxed{-16\sqrt[3]{3}}$

b) $\sqrt{6}\left(\sqrt{12} - \sqrt{3}\right)$

$= \sqrt{72} - \sqrt{18}$

$= \sqrt{2 \cdot 2 \cdot 2 \cdot 3 \cdot 3} - \sqrt{2 \cdot 3 \cdot 3}$

$= 6\sqrt{2} - 3\sqrt{2} = \boxed{3\sqrt{2}}$

c) $\left(6\sqrt{a} + \sqrt{3}\right)\left(2\sqrt{a} - \sqrt{4}\right)$ FOIL

$= 12\sqrt{a^2} - 6\sqrt{4a} + 2\sqrt{3a} - \sqrt{12}$

$= \boxed{12a - 12\sqrt{a} + 2\sqrt{3a} - 2\sqrt{3}}$

$\boldsymbol{a \geq 0}$

6. Rationalize each denominator.

a) $\frac{\sqrt{12}}{\sqrt{4}} \times \frac{\sqrt{4}}{\sqrt{4}}$

$= \frac{\sqrt{48}}{4} = \frac{\sqrt{2\cdot2\cdot2\cdot2\cdot3}}{4}$

$= \frac{\cancel{4}\sqrt{3}}{\cancel{4}} = \boxed{\sqrt{3}}$

b) $\frac{2}{(2+\sqrt{3})} \times \frac{(2-\sqrt{3})}{(2-\sqrt{3})}$

$=$ **$4 - 2\sqrt{3}$**

c) $\frac{(\sqrt{7}+\sqrt{28})}{(\sqrt{7}-\sqrt{14})} \times \frac{(\sqrt{7}+\sqrt{14})}{(\sqrt{7}+\sqrt{14})}$

$=$ **$-3 - 3\sqrt{2}$** 2

7. Solve the radical equation $(\sqrt{x+6})^2 = (x)^2$. Verify your anwer(s).

$x + 6 = x^2$

$0 = x^2 - x - 6$

$0 = (x-3)(x+2)$

$x = 3, -2$

Check.

$\sqrt{3+6} = 3$

$\sqrt{9} = 3$ ✓

$\sqrt{-2+6} = -2$

$\sqrt{4} = -2$ ✗

(reject)

$\boxed{x = 3}$

8. On a children's roller coaster ride, the speed in a loop depends on the height of the hill the car has just come down and the radius of the loop. The velocity, v, in feet per second, of a car at the top of a loop of radius r, in feet, is given by the formula $v = \sqrt{h-2r}$, where h is the height of the previous hill, in feet.

a) Find the height of the hill when the velocity at the top of the loop is 20 ft/s and the radius of the loop is 15 ft.

$V = 20$

$r = 15$

$h = ?$

$20 = \sqrt{h - 2(15)}$

$(20)^2 = (\sqrt{h-30})^2$

$400 = h - 30$

$430 = h$

height = 430 ft.

b) Would you expect the velocity of the car to increase or decrease as the radius of the loop increases? Explain your reasoning.

as $r \uparrow$ $v \uparrow$ as per equation.

$V = \sqrt{h - 2r}$

Chapter 6: Rational Expressions & Equations

9. Simplify each expression. Identify any non-permissible values.

a) $\dfrac{12\cancel{a^2}\cancel{b}}{48\cancel{a^2}b^{\cancel{4}3}}$

npv's: $a \neq 0,\ b \neq 0$

$\boxed{\dfrac{1}{4b^3}}$

b) $\dfrac{4-x}{x^2-8x+16}$

$-\dfrac{(\cancel{4-x})}{(\cancel{x-4})(x-4)}$

$x \neq 4$

$= \boxed{\dfrac{-1}{x-4}}$

c) $\dfrac{(x-3)(x+5)}{x^2-1} \div \dfrac{x+2}{x-3}$

$x \neq \pm 1,\ 3,\ -2$

$\dfrac{(x-3)(x+5)}{(x+1)(x-1)} \div \dfrac{x+2}{x-3}$

$\boxed{\dfrac{(x-3)(x+5)(x-3)}{(x+1)(x-1)(x+2)}}$

d) $\dfrac{5x-10}{6x} \times \dfrac{3x}{15x-30}$

$x \neq 0,\ 2$

$= \dfrac{\cancel{5}(\cancel{x-2})}{2\,\cancel{6x}} \times \dfrac{\cancel{3x}}{3\,\cancel{15}(\cancel{x-2})}$

$= \boxed{\dfrac{1}{6}}$

f) $\left(\dfrac{x+2}{x-3}\right)\left(\dfrac{x^2-9}{x^2-4}\right) \div \left(\dfrac{x+3}{x-2}\right)$

$x \neq \pm 2,\ 3,\ -3$

$= \dfrac{(\cancel{x+2})(x+3)(\cancel{x-3})}{(\cancel{x-3})(\cancel{x+2})(x-2)} \div \dfrac{(x+3)}{(x-2)}$

$= \dfrac{\cancel{x+3}}{\cancel{x-2}} \times \dfrac{\cancel{x-2}}{\cancel{x+3}} = \boxed{1}$

10. Determine the sum or difference. Express answers in lowest terms. Identify any non-permissible values.

a) $\dfrac{10}{a+2} + \dfrac{a-1}{a-7}$

$a \neq -2,\ 7$

$= \dfrac{10(a-7) + (a-1)(a+2)}{(a+2)(a-7)}$

$= \dfrac{10a - 70 + a^2 + 2a - a - 2}{(a+2)(a-7)}$

$= \boxed{\dfrac{a^2 + 11a - 72}{(a+2)(a-7)}}$

b) $\dfrac{3x+2}{x+4} - \dfrac{x-5}{x^2-4}$

TYPO

$x \neq 4,\ -4$

$= \dfrac{(3x+2)(x-4) - (x-5)(x+4)}{(x-4)(x+4)}$

$= \dfrac{3x^2 - 12x + 2x - 8 - x^2 - 4x + 5x + 20}{(x-4)(x+4)}$

$= \boxed{\dfrac{2x^2 - 9x + 12}{(x-4)(x+4)}}$

c) $\dfrac{2x}{x^2-25}-\dfrac{3}{x^2-4x-5}$ $x \neq \pm 5, -1$

$$\frac{2x}{(x+5)(x-5)} - \frac{3}{(x-5)(x+1)}$$

$$\frac{2x(x+1)-3(x+5)}{LCD} = \frac{2x^2+2x-3x-15}{LCD} = \frac{2x^2-x-15}{LCD} = \frac{(2x-6)(2x+5}{LCD}$$

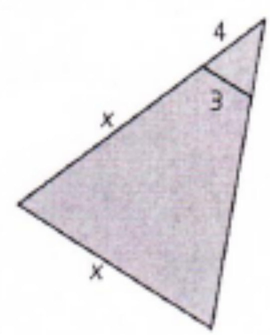
$$\boxed{= \frac{(x-3)(2x+5)}{(x+5)(x-5)(x+1)}}$$

11. Sandra simplified the expression $\dfrac{(x+2)(x+5)}{x+5}$ to $x+2$. She stated that they were equivalent expressions. Do you agree or disagree with Sandra's statement? Explain.

Not exactly. Must state $x \neq -2$

12. When two triangles are similar, you can use the proportion of corresponding sides to determine an unknown dimension. Solve the rational equation to determine the value of x.

$$\frac{x+4}{4} = \frac{x}{3}$$



$3(x+4) = x(4)$

$3x + 12 = 4x$

$\boxed{12 = x}$

Chapter 7: Absolute Value and Reciprocal Functions

13. Order the values from least to greatest.

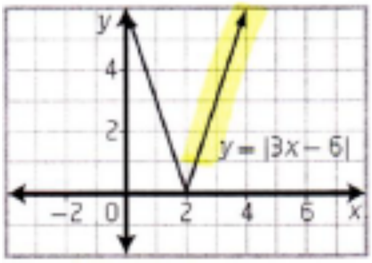
$|-5|, |4-6|, |2(-4)-5|, |8.4|$ $\quad -8-5$

5, 2, 13, 8.4

$2 < 5 < 8.4 < 13$

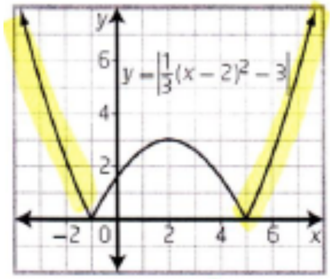
14. Write the piecewise function that represents each graph.

a)



$$y = \begin{cases} 3x - 6, & x \geq 2 \\ -3x + 6, & x < 2 \end{cases}$$

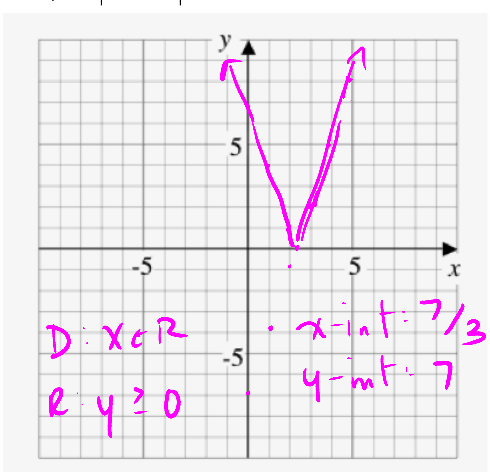
b)



$$y = \begin{cases} \frac{1}{3}(x-2)^2 - 3, & x \geq 5, x \leq -1 \\ -\frac{1}{3}(x-2)^2 + 3, & -1 < x < 5 \end{cases}$$

15. For each absolute value function,
i) sketch the graph, **ii)** determine the intercepts **iii)** determine the domain and range.

a) $y = |3x - 7|$

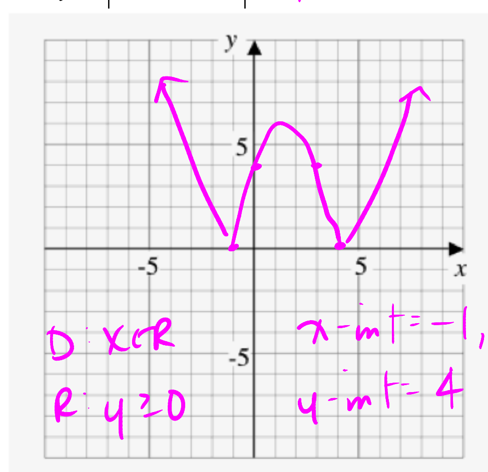


D: $x \in R$
R: $y \geq 0$

x-int: $7/3$
y-int: 7

b) $y = |x^2 - 3x - 4| = |(x-4)(x+1)|$

$|(-4)(1)|$
$= 4$



D: $x \in R$
R: $y \geq 0$

x-int: $-1, 4$
y-int: 4

16. Solve algebraically. Verify your solutions.

a) $|2x - 1| = 9$

Case +: $2x - 1 = 9$
$2x = 10$
$x = 5$

Case −: $-2x + 1 = 9$
$-2x = 8$
$x = 4$

Check $|10 - 1| = 9$ ✓

$\boxed{x = 5}$

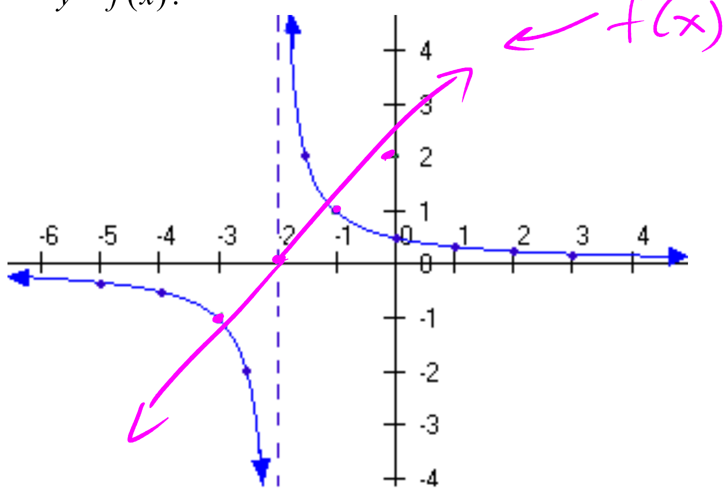
b) $|2x^2 - 5| = 13$

Case +: $2x^2 - 5 = 13$
$2x^2 = 18$
$x^2 = 9$
$x = \pm 3$

Case −: $-2x^2 + 5 = 13$
$-2x^2 = 8$
$x^2 = -4$
no solution

Check: $|2(3)^2 - 5| = 13$ ✓
$|2(-3)^2 - 5| = 13$ ✓

17. Sketch the graph of $y = f(x)$ given the graph of $y = \dfrac{1}{f(x)}$. What is the original function, $y = f(x)$?

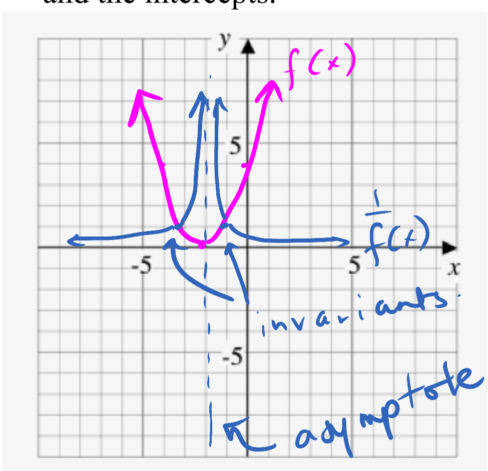


$m = 1$

$b = 2$

$f(x) = x + 2$

18. Sketch the graph of $y = \dfrac{1}{f(x)}$ given $f(x) = (x+2)^2$. Label the asymptotes, the invariant points, and the intercepts.



asymptote: $x = -2$

invariants: $(-3, 1)$, $(-1, 1)$

intercepts: $y = \frac{1}{4}$

19. Consider the function $f(x) = 3x - 1$.

a) What characteristics of the graph of $y = \dfrac{1}{f(x)}$ are different from those of $y = |f(x)|$?

Domain
Range
& Intercepts

$D: x \neq 1/3, x \in R$

$R: y \neq 0, y \in R$

$D: x \in R$

$R: y \geq 0$

b) Describe how the graph of $y = \dfrac{1}{f(x)}$ is different from the graph of $y = |f(x)|$.

Same