Chapter 1: Sequences & Series

1. Match each term to the correct expression.

I) arithmetic sequence ___A___
II) geometric sequence ___D___
III) arithmetic series ___E___
IV) geometric series ___C___
V) convergent series ___B___

~~A~~ 3,7,11,15,19,...
~~B~~ $5+1+\frac{1}{5}+\frac{1}{25}+...$
~~C~~ $1+2+4+8+16+...$
~~D~~ 1,3,9,27,81,...
~~E~~ $2+5+8+11+14$

2. Classify each sequence as arithmetic or geometric. State the value of the common difference or common ratio. Then, write the next three terms in each sequence.

a) 27,18,12,8,... $r=\frac{18}{27}=\frac{2}{3}$ $\frac{16}{3},\frac{32}{9},\frac{64}{27}$
Geometric

b) 17,14,11,8,... $d=14-17=-3$ $5,2,-1$
Arithmetic

c) −21,−16,−11,−6,... $d=-16-(-21)=5$ $-1,4,9$
Arithmetic

d) 3,−6,12,−24,... $r=\frac{-6}{3}=-2$ $48,-96,192$
Geometric

3. For each arithmetic sequence, determine the general term. Express your answer in simplified form. $t_n=t_1+(n-1)d$

a) 18,15,12,9,...
$d=-3$
$t_1=18$
$t_n=18+(n-1)(-3)$
$=18-3n+3$
$t_n=-3n+21$

b) $1,\frac{5}{2},4,\frac{11}{2},...$
$d=\frac{3}{2}$
$t_1=1$
$t_n=1+(n-1)(\frac{3}{2})$
$=1+\frac{3}{2}n-\frac{3}{2}$
$t_n=\frac{3}{2}n-\frac{1}{2}$

4. Use the general term to determine t_{20} in the geometric sequence $2,-4,8,-16,\ldots$.

$t_n = t_1 r^{n-1}$ $t_n = 2(-2)^{n-1}$ $t_{20} = 2(-2)^{20-1}$

$r = -2$

$t_1 = 2$

$t_{20} = -1048576$

5. **a)** What is S_{12} for the arithmetic series with a common difference of 3 and $t_{12} = 31$?

$t_1 = ?$ $31 = t_1 + (12-1)(3)$ $31 = t_1 + 33$ $t_1 = -2$

$S_n = \frac{n}{2}(t_1 + t_n)$ $S_{12} = \frac{12}{2}(-2+31)$

$S_{12} = 174$

b) What is S_5 for a geometric series where $t_1 = 4$ and $t_{10} = 78732$?

$S_n = \frac{t_1(r^n - 1)}{r-1}$ $78732 = 4(r)^9$ $\sqrt[9]{19683} = r$ $3 = r$

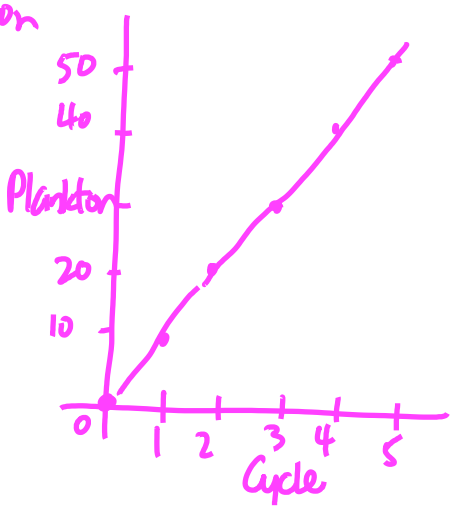
$S_5 = \frac{4(3^5 - 1)}{3-1}$

$S_5 = 484$

6. Phytoplankton, or algae, is a nutritional supplement used in natural health programs. Canadian Pacific Phytoplankton Ltd. is located in Nanaimo, British Columbia. The company can grow 10 t of marine phytoplankton on a regular 11 day cycle. Assume this cycle continues.

a) Create a graph showing the amount of phytoplankton produced for the first five cycles of production.

Cycle	Plankton
0	0
1	10
2	20
3	30
4	40
5	50



b) Write the general term for the sequence produced.

$t_1 = 0$ $d = 10$ $t_n = 0 + (n-1)(10)$

$t_n = 10n - 10$

7. The Living Shangri-La is the tallest building in Metro Vancouver. The ground floor of the building is 5.8 m high, and each floor above the ground floor is 3.2 m high. There are 62 floors altogether, including the ground floor. How tall is the building?

$t_1 = 5.8$

$d = 3.2$

$t_{62} = ?$ $t_{62} = 5.8 + (62-1)(3.2)$

$t_{62} = 201$ m

8. Tristan and Julie are preparing a math display for the school open house. Both students create posted to debate the following questions:

Does 0.999 … = 1 ?

Julie's Poster

0.999... ≠ 1
0.999... = 0.999 999 999 999 9...

The decimal will continue to infinity and will never reach exactly one.

Tristan's Poster

0.999... = 1
Rewrite 0.999... in expanded form.

$$\frac{9}{10}+\frac{9}{100}+\frac{9}{1000}+\cdots$$

This can be written as a geometric series where $t_1=\frac{9}{10}$ **and** $r=$

a) Finish Tristan's poster by determining the value of the common ratio and then finding the sum of the infinite geometric series.

$r=\frac{\frac{9}{100}}{\frac{9}{10}}=\frac{9}{100}\times\frac{10}{9}=\frac{1}{10}$

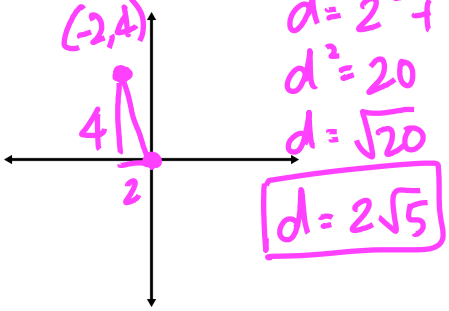
$S_\infty=\frac{t_1}{1-r}=\frac{\frac{9}{10}}{1-\frac{1}{10}}=\frac{\frac{9}{10}}{\frac{9}{10}}=\frac{9}{10}\times\frac{10}{9}=1$

b) Which student do you think correctly answered the question? Explain.

Tristan! At infinity it will reach 1

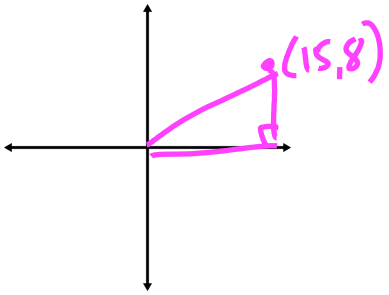
Chapter 2: Trigonometry

1. Determine the exact distance, in simplified form, from the origin to a point P (-2, 4) on the terminal arm of an angle.



$d^2=2^2+4^2$
$d^2=20$
$d=\sqrt{20}$
$d=2\sqrt{5}$

2. Point P (15,8) is on the terminal arm of angle θ. Determine the exact values for $\sin\theta$, $\cos\theta$ and $\tan\theta$



$d^2=8^2+15^2$
$d^2=289$
$d=17$

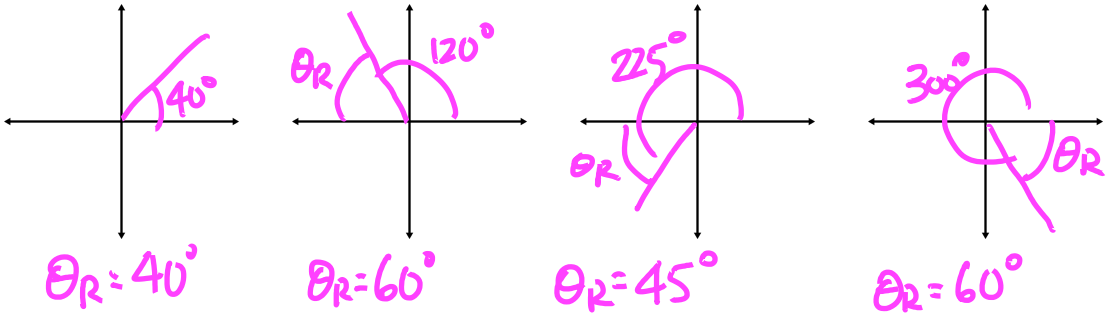
$\sin\theta=\frac{8}{17}$

$\cos\theta=\frac{15}{17}$

$\tan\theta=\frac{8}{15}$

3. Sketch each angle in standard position and determine the measure of the reference angle.

a) 40° **b)** 120° **c)** 225° **d)** 300°



$\theta_R = 40°$ $\theta_R = 60°$ $\theta_R = 45°$ $\theta_R = 60°$

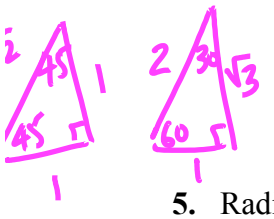
4. Determine the **exact** value of each trigonometric ratio.

a) $\sin 405°$ (+) $\theta_R = 45°$ $\sin 45° = \frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$

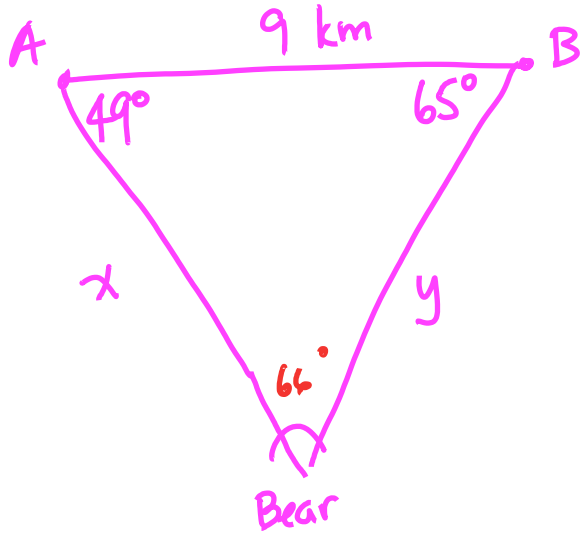
b) $\cos 330°$ (+) $\theta_R = 30°$ $\cos 30° = \frac{\sqrt{3}}{2}$

c) $\tan 225°$ (+) $\theta_R = 45°$ $\tan 45° = 1$

d) $\cos 150°$ (−) $\theta_R = 30°$ $\cos 30° = \frac{\sqrt{3}}{2} \rightarrow -\frac{\sqrt{3}}{2}$



5. Radio collars are used to track polar bears by sending signals via GPS to receiving stations. Two receiving stations are 9 km apart along a straight road. At station A, the signal from one of the collars comes from a direction of 49° from the road. At station B, the signal from the same collar comes from a direction of 65° from the road. Determine the distance the polar bear is from each of the stations.



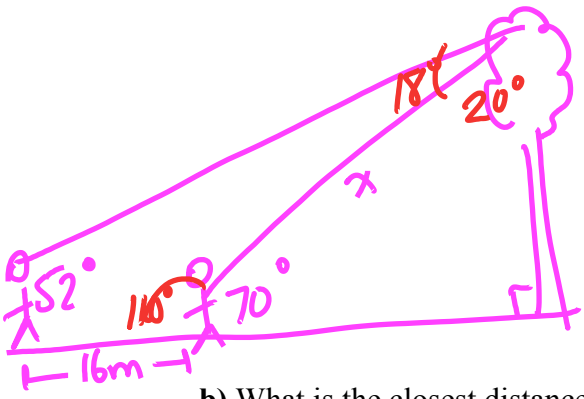
$\angle Bear = 66°$

$\frac{x}{\sin 65°} = \frac{9}{\sin 66°}$ $x = 8.9$ km

$\frac{y}{\sin 49°} = \frac{9}{\sin 66°}$ $y = 7.4$ km

6. Waterton Lakes National Park in Alberta is a popular site for birdwatching, with over 250 species of birds recorded. Chelsea spots a rare pileated woodpecker in a tree at an angle of elevation of 52°. After walking 16 m closer to the tree she determines the new angle of elevation to be 70°.

a) Sketch and label a diagram to represent the situation.

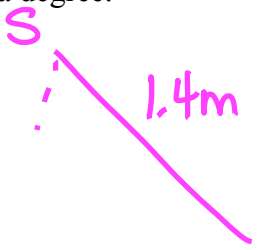


$\frac{16}{\sin 18^\circ} = \frac{x}{\sin 52^\circ}$

$x = 40.8\text{ m}$

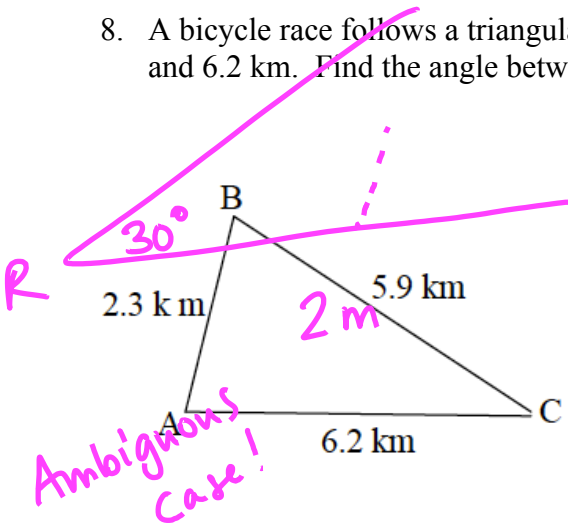
b) What is the closest distance that Chelsea is from the bird, to the nearest tenth of a meter?

7. In ΔRST, RT = 2 m, ST = 1.4 m, and $\angle R = 30°$. Determine the measure of obtuse $\angle S$ to the nearest tenth of a degree.



$\frac{\sin 30^\circ}{14} = \frac{\sin S}{2}$

8. A bicycle race follows a triangular course. The 3 legs of the race are in order 2.3 km, 5.9 km and 6.2 km. Find the angle between the starting leg and the finishing leg.



$$a^2 = b^2 + c^2 - 2bc(\cos A)$$
$$5.9^2 = 6.2^2 + 2.3^2 - 2(6.2)(2.3)(\cos A)$$
$$34.81 = 38.44 + 5.29 - 28.52\cos A$$
$$34.81 = 43.73 - 28.52\cos A$$
$$34.81 - 43.73 = -28.52\cos A$$
$$\frac{34.81 - 43.73}{-28.52} = \cos A$$
$$\therefore A = \cos^{-1}\left(\frac{-8.92}{-28.52}\right)$$
$$\therefore A \cong 72^o$$

$\therefore$ The angle between the starting leg and the finishing leg is $72°$.