

1. A rational number is of the form $\frac{a}{b}$, where a and b are integers.

a) What integer cannot be used for b ? Why?

$b \neq 0$, it would be undefined.

b) How does your answer to part a) relate to rational expressions? Explain using examples.

same idea, the terms that represent the denominator in a rational expression, could not be zero, otherwise it would be undefined.

2. What are the non-permissible values, if any, for each rational expression?

a) $\frac{5x^2}{2y}$

$y \neq 0$

b) $\frac{x^2 - 1}{x + 1}$

$x + 1 \neq 0$
 $x \neq -1$

c) $\frac{27x^2 - 27}{3}$

no non-permissible value.

d) $\frac{7}{(a - 3)(a + 2)}$

$a - 3 \neq 0$, $a + 2 \neq 0$
 $a \neq 3$, $a \neq -2$

e) $\frac{-3m + 1}{2m^2 - m - 3}$

$(2m - 3)(m + 1)$
 $2m - 3 \neq 0$, $m + 1 \neq 0$
 $m \neq \frac{3}{2}$, $m \neq -1$

3. What is the numerical value for each rational expression? Test your result using some permissible values for the variable. Identify any non-permissible values.

a) $\frac{2s - 8s}{s}$

$= \frac{-6s}{s} = -6$

try $s = 2$

$\frac{4 - 16}{2} = \frac{-12}{2} = -6$ ✓

b) $\frac{5x - 3}{3 - 5x} = \frac{(5x - 3)}{-(5x - 3)}$

$= -1$

try $x = 2$

$\frac{10 - 3}{3 - 10} = \frac{7}{-7} = -1$ ✓

c) $\frac{2 - b}{4b - 8} = \frac{-(b - 2)}{4(b - 2)}$

$= \frac{-1}{4}$

try $b = 3$

$\frac{2 - 3}{12 - 8} = \frac{-1}{4}$ ✓

4. Write an expression that satisfies the given conditions in each case.

a) equivalent to $\frac{x-3}{5}$, with a denominator of $10x$

$$\frac{(x-3)(2x)}{5(2x)} \rightarrow \frac{2x^2-6x}{10x}$$

b) equivalent to $\frac{x-3}{x^2-9}$, with a numerator of 1

$$\frac{\cancel{x-3}}{(\cancel{x-3})(x+3)} \rightarrow \frac{1}{x+3}$$

c) equivalent to $\frac{c-2d}{3f}$, with a numerator of $3c-6d$

$$\frac{3 \cdot (c-2d)}{3 \cdot (3f)} \rightarrow \frac{3c-6d}{9f}$$

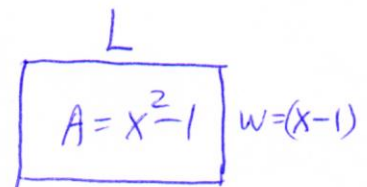
d) equivalent to $\frac{m+1}{m+4}$, with non-permissible values of ± 4

$$\frac{(m+1)(m-4)}{(m+4)(m-4)} \rightarrow \frac{(m+1)(m-4)}{(m+4)(m-4)} \text{ or } \frac{m^2-3m-4}{m^2-16}$$

5. A rectangle has area $x^2 - 1$ and width $x - 1$.

a) What is a simplified expression for the length?

$$\begin{aligned} L &= \frac{A}{w} = \frac{(x^2-1)}{(x-1)} \\ &= \frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}} \\ &= (x+1) \end{aligned}$$



b) Identify any non-permissible values. What do they mean in this context?

$$\boxed{x \neq 1} \text{ which would make the width } = 0$$

$$\boxed{x \neq -1} \text{ which would make the length } = 0$$

6. Simplify each product. Determine all non-permissible values.

$$a) \frac{2p}{r} \times \frac{10q}{8p} = \frac{5q}{2r}$$

$$\boxed{r \neq 0, p \neq 0}$$

$$b) 4m^2 \times \frac{1}{16mt^3} = \frac{m^2}{4t^3}$$

$$\boxed{m \neq 0, t \neq 0}$$

$$c) \frac{3a+3b}{8} \times \frac{4}{a+b} = \frac{3}{2}$$

$$\boxed{a \neq -b}$$

$$d) \frac{x^2-4}{x^2+25} \times \frac{2x^2+10x}{x^2+2x}$$

$$= \frac{(x-2)(x+2)}{x^2+25} \cdot \frac{2x(x+5)}{x(x+2)}$$

$$= \frac{2(x-2)(x+5)}{x^2+25}, \quad \boxed{x \neq 0, -2}$$

$$e) \frac{d^2+3d+2}{2d+2} \times \frac{2d+6}{d^2+5d+6}$$

$$= \frac{(d+2)(d+1)}{2(d+1)} \cdot \frac{2(d+3)}{(d+3)(d+2)} = 1, \quad \boxed{d \neq -1, -2, -3}$$

$$f) \frac{y^2-8y-9}{y^2-10y+9} \times \frac{y^2-9y+8}{y^2-1} \times \frac{y^2-25}{5-y}$$

$$= \frac{(y-9)(y+1)}{(y-9)(y-1)} \cdot \frac{(y-8)(y-1)}{(y-1)(y+1)} \cdot \frac{(y-5)(y+5)}{-1(y-5)} = \frac{-(y-8)(y+5)}{(y-1)}$$

$$\boxed{y \neq -1, 1, 5, 9}$$

7. Multiply or divide as indicated. Express answers in simplest form. Determine all non-permissible values.

$$a) \frac{x^2-3x+2}{x^2-4} \times \frac{x+3}{x^2+3x} \div \frac{1}{x+2} = \frac{(x-2)(x-1)}{(x-2)(x+2)} \cdot \frac{(x+3)}{x(x+3)} \cdot \frac{(x+2)}{1}$$

$$= \frac{x-1}{x}, \quad \boxed{x \neq 0, \pm 2, -3}$$

$$b) \frac{a-3}{a-4} \div \frac{30}{a+3} \times \frac{5a-20}{a^2-9} = \frac{a-3}{a-4} \cdot \frac{a+3}{30} \cdot \frac{5(a-4)}{(a-3)(a+3)}$$

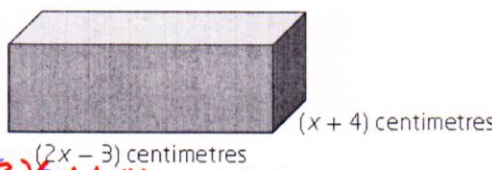
$$= \frac{1}{6}, \quad \boxed{a \neq \pm 3, 4}$$

8. The volume of a rectangular prism is $(2x^3 + 5x^2 - 12x) \text{ cm}^3$. If the length of the prism is $(2x - 3) \text{ cm}$ and its width is $(x + 4) \text{ cm}$, what is an expression for the height of the prism?

$V = L \cdot W \cdot h$

$h = V \div L \div W$

$h = \frac{2x^3 + 5x^2 - 12x}{(2x - 3)(x + 4)} = \frac{x(2x^2 + 5x - 12)}{(2x - 3)(x + 4)} = \boxed{x} \text{ cm}$



9. Add or subtract. Express answers in simplest form. Identify any non-permissible values.

a) $\frac{4x - 3}{6} - \frac{x - 2}{4} = \frac{8x - 6 - 3x + 6}{12} = \frac{5x}{12}$, No non-permissible value.

b) $\frac{9}{x - 3} + \frac{7}{x^2 - 9} = \frac{9(x + 3) + 7}{(x - 3)(x + 3)} = \frac{9x + 34}{(x - 3)(x + 3)}$

$x \neq \pm 3$

c) $\frac{a}{a - b} - \frac{2ab}{a^2 - b^2} + \frac{b}{a + b}$

$= \frac{a(a + b) - 2ab + b(a - b)}{(a - b)(a + b)} = \frac{a^2 + ab - 2ab + ab - b^2}{(a - b)(a + b)}$

$a \neq \pm b$

$= \frac{a^2 - b^2}{a^2 - b^2} = 1$

d) $\frac{2x}{4x^2 - 9} + \frac{(2x + 3)(x + 1)}{2x^2 + 5x + 3} - \frac{1}{2x - 3}$

$= \frac{2x(x + 1) + x(2x - 3) - (2x + 3)(x + 1)}{(2x - 3)(2x + 3)(x + 1)}$ $\rightarrow 2x^2 + 5x + 3$

$= \frac{2x^2 + 2x + 2x^2 - 3x - 2x^2 - 5x - 3}{(2x - 3)(2x + 3)(x + 1)} = \frac{2x^2 - 6x - 3}{(2x - 3)(2x + 3)(x + 1)}$

$x \neq \pm \frac{3}{2}, -1$

10. Solve each rational equation. Identify all non-permissible values.

a) $\frac{s-3}{s+3} = 2$

$s-3 = 2s+6$
 $-9 = s$

$s \neq -3$

b) $\frac{z-2}{z} + \frac{1}{5} = \frac{-4}{5z}$

LCD = 5z $\xrightarrow{\text{LCD}}$ $\frac{5z-10+z}{5z} = \frac{-4}{5z}$ ~~LCD~~

$6z-10 = -4$, $6z = 6$ $\boxed{z=1}$ $\boxed{z \neq 0}$

c) $\frac{3m}{m-3} + 2 = \frac{3m-1}{m+3}$

LCD = (m-3)(m+3)

~~LCD~~ $\frac{3m(m+3) + 2(m-3)(m+3)}{(m-3)(m+3)} = \frac{(3m-1)(m-3)}{(m-3)(m+3)}$ ~~LCD~~

$m \neq \pm 3$

$3m^2+9m+2m^2-18 = 3m^2-10m+3$
 $2m^2+19m-21 = 0$
 $(2m+21)(m-1) = 0$

$m = -\frac{21}{2}$ or $m = 1$

d) $\frac{x}{x-3} = \frac{3}{x-3} - 3$

~~LCD~~ $\frac{x}{x-3} = \frac{3-3(x-3)}{x-3}$ $\xrightarrow{\text{LCD}}$ $x = 3-3x+9$
 $4x = 12$

$\boxed{\text{No solution}}$

$\boxed{x=3}$ rejected
 non-permissible
 value.
 $\boxed{x \neq 3}$

e) $\frac{3}{x+2} + \frac{5}{x-3} = \frac{3x}{x^2-x-6} - 1$

$(x-3)(x+2)$

$\xrightarrow{\text{LCD}}$ $\frac{3(x-3)+5(x+2)}{(x+2)(x-3)} = \frac{3x-x^2+x+6}{(x+2)(x-3)}$ ~~LCD~~

$\boxed{x \neq -2, 3}$

$3x-9+5x+10 = -x^2+4x+6$

$x^2+4x-5 = 0$

$(x+5)(x-1) = 0$

$\boxed{x = -5}$
 $\boxed{x = 1}$

11. Matt and Elaine, working together, can paint a room (in 3 h). It would take Matt 5 h to paint the room by himself. How long would it take Elaine to paint the room by herself?

$T_{\text{together}} = 3 = \frac{1}{(r_M + r_E)}$

$r_M + r_E = \frac{1}{3}$, $r_M = \frac{1}{5}$

$r_E = \frac{1}{t_E} \Rightarrow t_E = \frac{1}{r_E}$

$r_E = \frac{1}{3} - \frac{1}{5}$

$r_E = \frac{5-3}{15} = \frac{2}{15}$

$t_E = \frac{1}{\frac{2}{15}} = \frac{15}{2} = \boxed{7.5 \text{ h}}$

12. An elevator goes directly from the ground up to the observation deck of the Calgary Tower, which is at 160 m above the ground. The elevator stops at the top for 36 s before it travels directly back down to the ground. The time for the round trip is 2.5 min. The elevator descends at 0.7 m/s faster than it goes up.

a) Determine an equation that could be used to find the rate of ascent of the elevator.

Let U be the speed of going up.

	d	U	t
up	160	U	$\frac{160}{U}$
down	160	$U+0.7$	$\frac{160}{U+0.7}$

$$t = 2.5 \text{ min} = 150 \text{ sec} - 36 \text{ sec}$$

$$t_{\text{total}} = 114 \text{ sec}$$

traveling

b) Simplify your equation to the form $ax^2 + bx + c = 0$, where a , b , and c are integers, and then solve.

$$\frac{160}{U} + \frac{160}{U+0.7} = 114 \Rightarrow \frac{160(U+0.7) + 160U}{U(U+0.7)} = 114 \frac{U}{(U+0.7)}$$

$$\Rightarrow 160U + 112 + 160U = 114U^2 + 79.8U$$

$$(0 = 114U^2 - 240.2U - 112) \cdot 5 \quad \text{to eliminate the decimal}$$

$$0 = 570U^2 - 1201U - 560$$

$$U = \frac{1201 \pm \sqrt{(-1201)^2 - 4(570)(-560)}}{2(570)} = \frac{1201 \pm 1649}{1140} \begin{matrix} \nearrow \text{(i)} \\ \searrow \text{(ii)} \end{matrix}$$

c) What is the rate of ascent in km/h, to the nearest tenth?

$$\text{(i)} \quad 2.5 \frac{\text{m}}{\text{s}} = \frac{2.5 \div 1000}{1 \div 3600} = 9 \frac{\text{km}}{\text{h}}$$

~~(ii) $-0.3929... \frac{\text{m}}{\text{s}}$~~ reject, speed can not be negative.