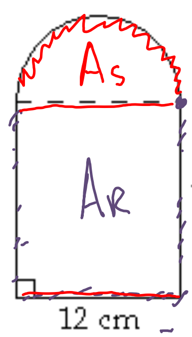
Lesson 2.1 Surface Area & Volume – Perimeter & Area of 2-Dimensional Figures

In this lesson, we will use the following formulas to calculate the perimeter and area of different figures.

Circle:	$C = \pi d$ or $C = 2\pi r$	$A = \pi r^2$
Square:	$P = 4 \cdot l$	$A = l^2$
Rectangle:	$P = 2l + 2w$ or $P = 2(l + w)$	$A = l \cdot w$
Triangle:	$P = a + b + c$	$A = \frac{1}{2} b \cdot h$ or $A = \frac{b \cdot h}{2}$
Parallelogram:	$P =$ sum of all sides	$A = b \cdot h$
Trapezoid:	$P =$ sum of all sides	$A = \frac{1}{2}(b_1 + b_2) \cdot h$ or $A = \frac{(b_1 + b_2) \cdot h}{2}$

Eg1. Find the perimeter and area of the figures below. Express answers in imperial.

a)



$18\text{ cm} \times \frac{1\text{ in}}{2.54\text{ cm}} = 7.1\text{ in}$

$12\text{ cm} = 4.7\text{ in}$

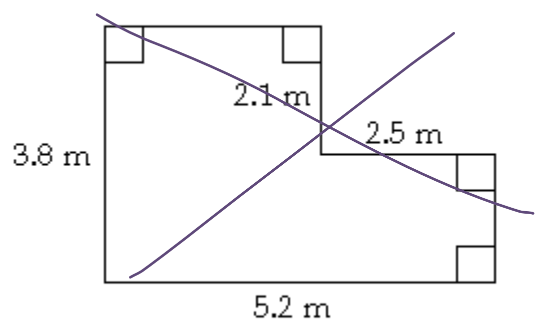
P = distance around

$P = 7.1(2) + 4.7 + \underline{7.4}$

$\frac{C}{2} = \pi d/2 = \frac{\pi(4.7)}{2} = 7.4$

$\boxed{P = 26.3\text{ in}}$

b)



$r = \frac{d}{2} = \frac{4.7}{2} = 2.35$

$A_R = lw = 7.1 \times 4.7 = 33.37\text{ in}$

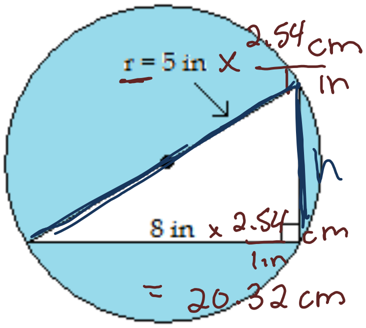
$A_S = \frac{\pi r^2}{2} = \frac{\pi(2.35)^2}{2} = 8.7\text{ in}$

$A_T = A_R + A_S$

$= 33.37 + 8.7$

$\boxed{A_T = 42.07\text{ in}^2}$

Eg2. Find the area of the shaded region. Express the answer using metric units.



$r = 5\text{ in} \times \frac{2.54\text{ cm}}{1\text{ in}} = 12.7\text{ cm}$

$8\text{ in} \times \frac{2.54\text{ cm}}{1\text{ in}} = 20.32\text{ cm}$

$h:\ a^2 + b^2 = c^2$

$20.32^2 + h^2 = 25.4^2$

$h^2 = 25.4^2 - 20.32^2$

$\sqrt{h^2} = \sqrt{232.3}$

$h = 15.24\text{ cm}$

$A_C = \pi r^2$

$= \pi(12.7)^2$

$A_C = 506.7\text{ cm}^2$

$A_T = \frac{bh}{2} = \frac{(20.32)(15.24\text{ cm})}{2}$

$A_T = 154.8\text{ cm}^2$

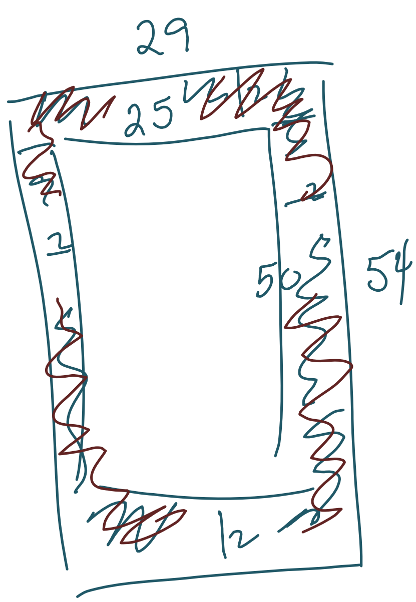
$A_S = A_C - A_T$

$= 506.7 - 154.8$

$\boxed{A_S = 351.9\text{ cm}^2}$

Eg3. A minimum of 2-metre wide sidewalk is to be built for an Olympic-size swimming pool (50 m long and 25 m wide). How many tiles are needed to decorate the sidewalk if each tile is 1-foot by 1-foot?

$29\text{ m} \times \frac{100\text{ cm}}{1\text{ m}} \times \frac{1\text{ ft}}{30.48\text{ cm}} = 95.1\text{ ft}$



$25\text{m} = 82\text{ ft}$

$50\text{m} = 164\text{ ft}$

$54\text{m} = 177.2\text{ ft}$

$A_L = lw = (95.1)(177.2)$

$A_L = 16851.72\text{ ft}^2$

$A_S = lw = (82)(164)$

$= 13448\text{ ft}^2$

$A = A_L - A_S$

$= 16851.72 - 1344\ldots$

$\cancel{A = 3403.72}\text{ ft}^2$

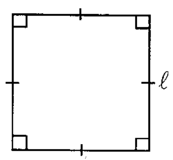
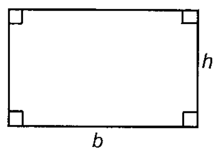
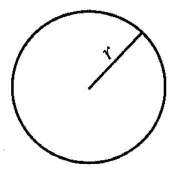
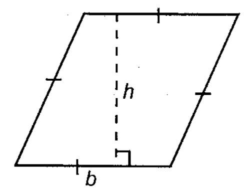
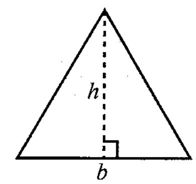
$\boxed{3404\text{ tiles}}$

Practices: Perimeter & Area Worksheet

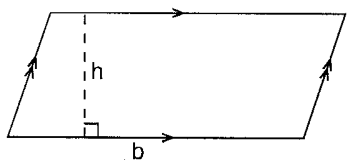
Perimeter & Area Worksheet

Area of rectangles, squares, circles, parallelograms, triangles and trapezoids

Review of area formulas:

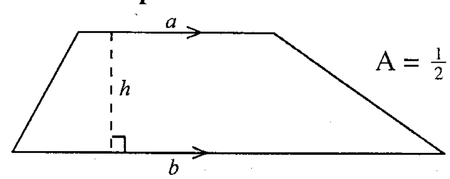
Square	Rectangle	Circle	Rhombus	Triangle
				
$A = \ell^2$	$A = b \cdot h$	$A = \pi r^2$	$A = b \cdot h$	$A = \frac{1}{2}bh$
$P = 4\ell$	$P = 2b + 2h$	$C = 2\pi r$		

Parallelogram



$A = b \cdot h$

Trapezoid

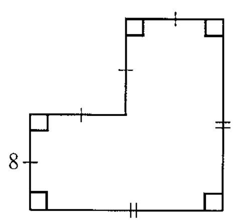


$A = \frac{1}{2}h(a+b)$

EXERCISE

Find the area of the following figures or shaded regions. All measurements are in centimetres.

1.



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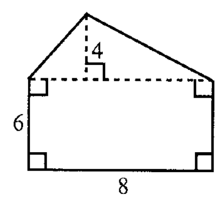
2.



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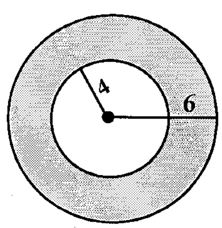
3.



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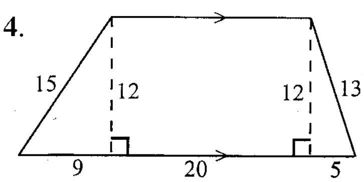

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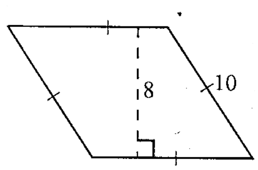
5.



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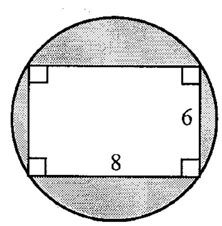
6.



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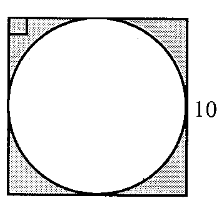
7.



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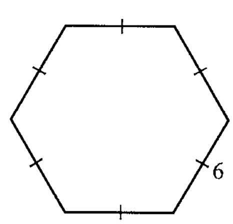
8.



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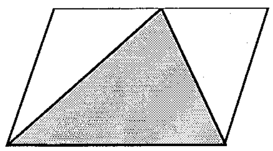
APPLICATIONS

1. 

The shaded area is what fraction of the area of the parallelogram?

2. Ann wants to wallpaper the walls of her bedroom. The rectangular room is 12 ft. by 15 ft. with height of the wall 8 ft. high. If a roll of wallpaper is 3 feet wide and 50 feet long, how many rolls of wallpaper are needed? (*can't buy part of a roll*)

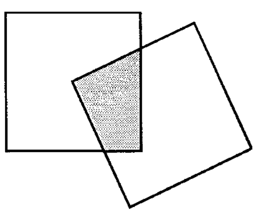
3. If it takes 2 minutes to walk around a circular garden, how long does it take, to the nearest second, to go through the diameter of the garden?

4. A circular swimming pool with a diameter of 24 *m* has a brick path 1 *m* wide surrounding the pool. Calculate the surface area of the brick path.

5. A rectangular lawn measuring 30 *m* by 18 *m* has 3 circular flowerbeds cut from it of diameter 3 *m* each. Find the area of the grass to the nearest square metre.

6. An isosceles trapezoid with bases 12 *cm* and 28 *cm* has an area of 300 cm^2.

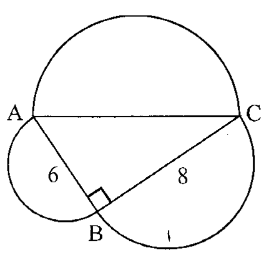
(**a**) find the height (**b**) find the perimeter.

7. 

Two squares each with side 10 *cm* are placed such that a vertex of one lies at the centre of the other.

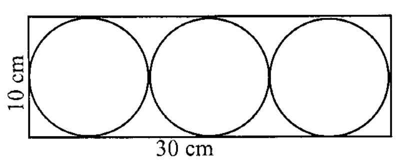
What is the area of shaded region?
(*Hint: rotate square to better position*)

8. A bike with wheels of radius 30 *cm* makes 20 revolutions. How far does the bike travel in metres?

9. 

Semi-circles are constructed on the sides of right triangle ABC. Find the total area of the semi-circles.

10. Jessica wants to tile the kitchen floor with tiles that are 20 *cm* square. If the kitchen measures 2.5 *m* by 3 *m*, how many tiles are needed if 10% is added for waste?

11. 

Three metal discs are to be cut from a piece of metal 30 *cm* long by 10 *cm* wide.

a) How much metal is wasted?
b) What percent of the metal is wasted?

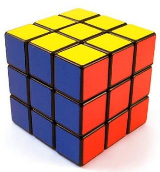
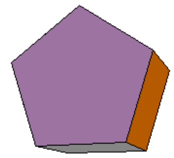
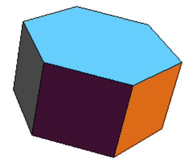
12. Andy has a contract to paint the walls and ceiling of 30 motel rooms measuring 3.5 *m* by 4.2 *m* with height of 2.1 *m*. If a 4-litre pail of paint covers 40 m^2 and cost $18.95, what is the cost of paint used?

p.1 Ans: 1) 192 cm^2 2) 64 cm^2 3) 62.83 cm^2 4) 324 cm^2 5) 80 cm^2 6) 30.54 cm^2 7) 21.46 cm^2 8) 93.53 cm^2
p.2 Ans: 1) ½ the size 2) 3 rolls 3) 38.2 seconds 4) 78.54 m^2 5) 518.79 m^2 6a) 15 cm 6b) 74 cm
7) 25 cm^2 8) 37.70 m 9) 78.54 unit^2 10) 207 tiles 11a) 64.38 cm^2 11b) 21.46% 12) $682.20

Lesson 2.2 Surface Area & Volume – Prisms & Cylinders

A prism is a three-dimensional object with both the base and the cap having the same shape and size. A right prism is a prism with all lateral sides perpendicular to the base/cap.

Some examples of right prism are:

Cube	Cylinder	Triangular Prism	Rectangular Prism	Pentagonal Prism	Hexagonal Prism
					

The surface area of any prism is: sum of areas on all surfaces

The volume of any prism is: base area × height

Eg1. Consider the lateral side of a cylinder. Give the name of its shape when you lay it flat.

Rectangle

Eg2. How many surfaces are on the following shapes?

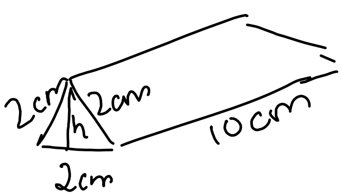
a) cube 6

b) triangular prism 5

c) rectangular prism 6

d) hexagonal prism 8

Eg3. A 10 cm long triangular prism has equilateral triangles at both ends. If the equilateral triangles have side length 2 cm, determine the volume and surface area of the prism.



$SA = 2A_T + 3A_R$

$= 2\left(\frac{bh}{2}\right) + 3(lw)$

$= 2\left(\frac{\cancel{2}\cdot\sqrt{3}}{\cancel{2}}\right) + 3(2)(10)$

$\boxed{SA = 63.46\ cm^2}$

$a^2 + b^2 = c^2$

$1^2 + h^2 = 2^2$

$h^2 = 3$

$h = \sqrt{3}$

length

$V = \left(\text{area of base}\right)h$

$= \left(\frac{bh}{2}\right)l$

$= \left(\frac{\cancel{2}\sqrt{3}}{\cancel{2}}\right)(10)$

$\boxed{V = 17.32\ cm^3}$

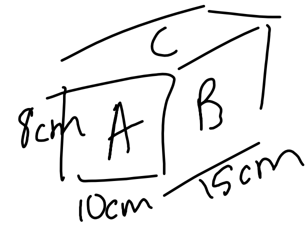
Eg4. A packing company use corrugated cardboard for containers and ship parcels to the United States. A container is 15 cm long, 10 cm deep, and 8 cm high.

a) Find its volume.

$V = lwh$

$= 15(10)(8)$

$\boxed{V = 1200\ cm^3}$



b) Express the surface area in inches.

$(2.54\,cm)^2 = (1\,in)^2$

$6.4516\,cm^2 = 1\,in^2$

$SA = (A_A + A_B + A_C)2$

$= [(8)(10) + (8)(15) + (10)(15)]2$

$SA = 700\,\cancel{cm^2} \times \frac{1\,in^2}{6.4516\,\cancel{cm^2}} = \boxed{108.5\,in^2}$

c) Calculate the cost of the container if the corrugated cardboard is \$0.20/in^2.

$108.5\,\cancel{in^2} \times \frac{\$0.20}{1\,\cancel{in^2}} = \boxed{\$21.70}$

$42\,\cancel{gal} \times \frac{3785.41\,cm^3}{1\,\cancel{gal}}$

$= 158987.29\ cm^3$

= 88cm convert all units to cm!!

Eg6. A barrel of crude oil is 880 mm tall. Assuming the barrel is completely filled with 42 gallons of oil and has no empty space, calculate the radius of the barrel in inches.

Info required: 1 gallon = 3785.41178 cm^3

$V = \pi r^2 h$

$\sqrt{r^2} = \sqrt{\frac{V}{\pi h}}$

$r = \sqrt{\frac{V}{\pi h}}$

$= \sqrt{\frac{158987.2935}{\pi(88)}}$

$= 23.98\ cm$

$= 23.98\,\cancel{cm} \times \frac{1\,in}{2.54\,\cancel{cm}}$

$= \boxed{9.44\,in}$

Practices: Textbook p.61 # 4, 5, 10

p.74 # 1ab, 3ab, 4, 6, 8, 17

p.86 # 3, 5ac, 7, 13

Lesson 2.3 Surface Area & Volume – Pyramids, Cones & Spheres

Any solid that has a base and comes to a point (apex) is called a pyramid.

The surface area of a pyramid consists of:

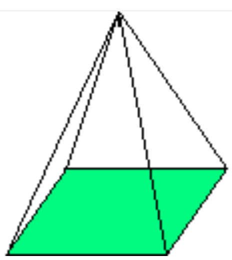
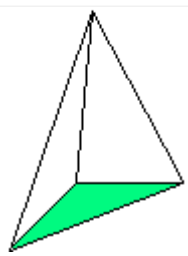
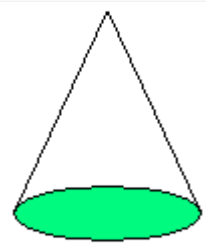
→ the base area, and

→ the area of all the lateral sides

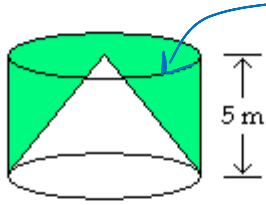
Whereas the volume of a pyramid is calculated using:

→ Vol = $\frac{1}{3}$ (Base Area x Height)

Common Types of Pyramids:

			
	rectangular pyramid	triangular pyramid	cone
	SA = base + 4 x triangles	SA = base + 3 x triangles	SA = base + πrs
	Vol = $\frac{1}{3}$(BA x h)	Vol = $\frac{1}{3}$(BA x h)	Vol = $\frac{1}{3}$(BA x h)

Eg2. The height and the radius for the following solid are of the same measure. Find the volume that is inside the cylinder but outside the cone.



Make sense of the formula's !!

$V = \frac{2}{3} V_c$ ← cylinder

$= 2/3\ (\pi r^2 h)$

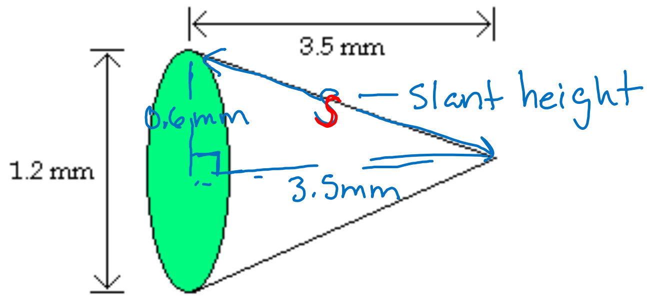
$= 2/3\ (\pi (5)^3)$

$= \boxed{261.8\ m^3}$

Cones:

Eg3. Find the volume and the surface area of the following pyramid.

a)



$SA = \pi r^2 + \pi r s$

$= \pi(0.6)^2 + \pi(0.6)(3.55)$

$= \boxed{7.82 mm^2}$

$a^2 + b^2 = c^2$

$0.6^2 + 3.5^2 = s^2$

$\sqrt{12.61} = \sqrt{s^2}$

$3.55 = s$

$V = \frac{1}{3}(\text{area of base} \times \text{height})$

$= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi(0.6)^2(3.5)$

$= \boxed{1.32 mm^3}$

Spheres:

A sphere is a perfectly round three-dimensional object.

The volume of a sphere is four-thirds the volume of a cylinder with the same radius and height ➔ $V = \frac{4}{3}\pi r^3$

Its surface area is simply (orange demo) ➔ $SA = 4\pi r^2$

Eg1. Determine the missing parameter for each of the following sphere.

a) Volume = 50 m³, find r

$V = \frac{4}{3}\pi r^3$

$3(50) = \left(\frac{4}{\cancel{3}}\pi r^3\right)\cancel{3}$

$\frac{150}{4\pi} = \frac{\cancel{4\pi} r^3}{\cancel{4\pi}}$

$\sqrt[3]{\frac{150}{4\pi}} = \sqrt[3]{r^3}$

$\boxed{r = 2.29 m}$

b) Circumference = 2.8 ft, find SA

$C = 2\pi r$

$\frac{2.8}{2\pi} = \frac{\cancel{2\pi} r}{\cancel{2\pi}}$

$0.45 = r$

$SA = 4\pi r^2$

$= 4\pi(0.45)^2$

$= \boxed{2.5 ft^2}$

Eg3. Three tennis balls of radius 6.7 cm fit tightly in a cylindrical container. How much space is unoccupied in the container?

$3 V_s = 3\left(\frac{4}{3}\pi r^3\right)$

$$V_U = \pi r^2 h - 4\pi r^3$$

$$V_U = \pi r^2(6r) - 4\pi r^3$$

$$V_U = 6\pi r^3 - 4\pi r^3$$

$$V_U = 2\pi r^3$$

$$V_U = 2\pi(6.7)^3$$

$$\boxed{V_U = 1889.75 \text{ cm}^3}$$

$$V_{3S} = 4\pi r^3$$

$$V_C = \pi r^2 h$$

Practices: Textbook p.74 # 1cde, 3c, 10, 12, 14
Textbook p.86 # 1ad, 2, 5d, 8, 11