Pre-Calculus 11 Ch# 8 Test Review Name: KEY

Date:__________________ Block:_______

Where necessary, round your answers to the nearest hundredth.

1. Consider the tables of values for $y = -1.5x - 2$ and $y = -2(x-4)^2 + 3$.

a) Use the tables to determine a solution to the system of equations
$y = -1.5x - 2$
$y = -2(x-4)^2 + 3$

b) Verify this solution by graphing.

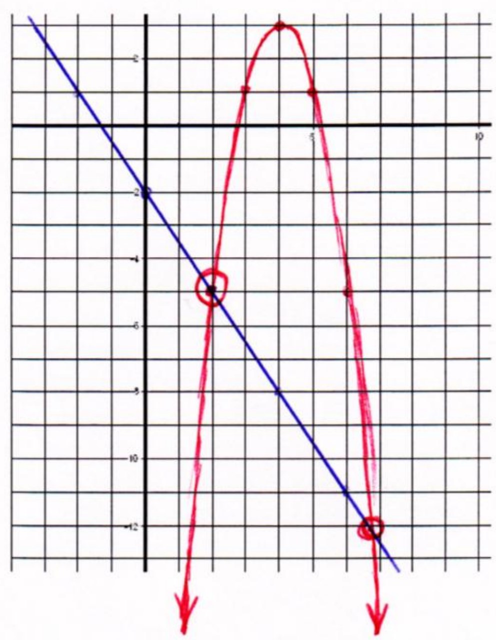
c) What is the other solution to the system?

x	y
0.5	−2.75
1	−3.5
1.5	−4.25
2	−5
2.5	−5.75
3	−6.5

x	y
0.5	−21.5
1	−15
1.5	−9.5
2	−5
2.5	−1.5
3	1

a) solution: $(2, -5)$

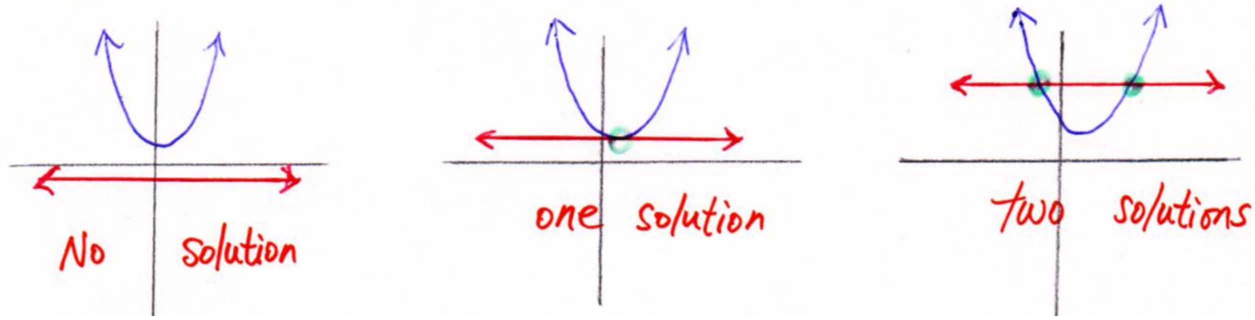
b)



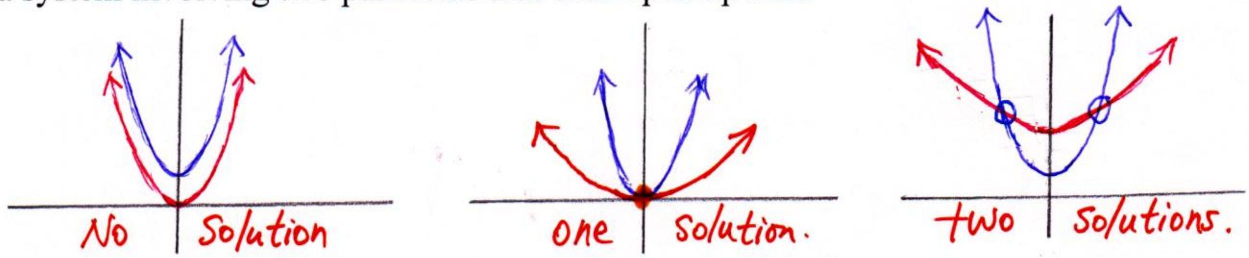
c) According to the graph. the other solution is approximately located at : $(6.8, -12)$

2. State the number of possible solutions to each system. Include sketches to support your answers.

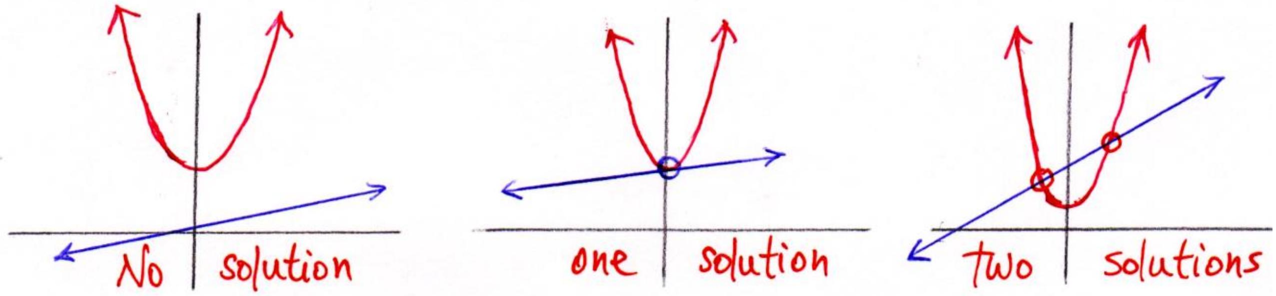
a) a system involving a parabola and a horizontal line



b) a system involving two parabolas that both open upward



c) a system involving a parabola and a line with a positive slope



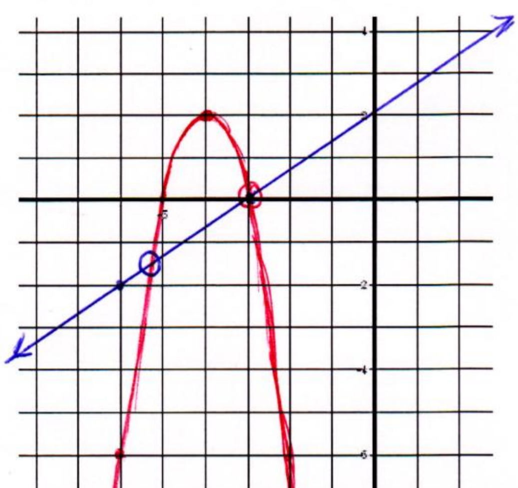
3. Solve each system of equations by graphing.

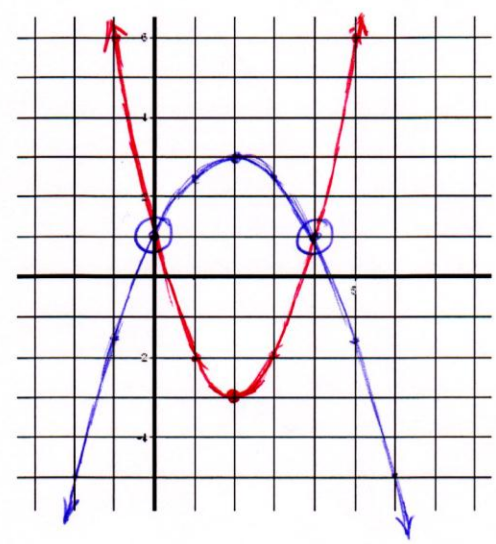
a) $y = \frac{2}{3}x + 2$

$y = -2(x + 4)^2 + 2$

b) $y = x^2 - 4x + 1$

$y = -\frac{1}{2}(x - 2)^2 + 3$





Solutions:
$(-3, 0)$
approximately $(-5.2, -1.5)$

Solutions: $(0, 1)$
$(4, 1)$

4. Adam graphed the system of quadratic equations $y = x^2 + 1$ and $y = x^2 + 3$ on a graphing calculator. He speculates that the two graphs will intersect at some large value of y. Is Adam correct? Explain.

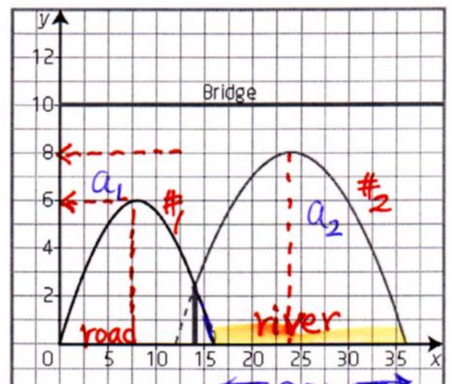
NO, Adam is not correct, because these 2 parabolas are of the same size and x^2+1 is 2 units above x^2+3, which create a condition that they never intersect.

5. An engineer constructs side-by-side parabolic arches to support a bridge over a road and a river. The arch over the road has a maximum height of 6 m and a width of 16 m. The river arch has a maximum height of 8 m, but its width is reduced by 4 m because it intersects the arch over the road. Without this intersection, the river arch would have a width of 24 m. A support footing is used at the intersection point of the arches. The engineer sketched the arches on a coordinate system. She placed the origin at the left most point of the road.

a) Determine the equation that models each arch.

b) Solve the system of equations.

c) What information does the solution to the system give the engineer?



$V_1:(7.5, 6)$; $V_2:(23.75, 8)$

$\approx(8, 6)$; $\approx(24, 8)$

2.5

1

a).

$0 = a_1(x-8)^2+6$

$-6 = 64 \cdot a_1 \Rightarrow a_1 = \frac{-6}{64} = \frac{-3}{32}$

Equation #1 : $y = \frac{-3}{32}(x-8)^2+6$

$0 = a_2(36-24)^2+8$

$-8 = 144a_2 \Rightarrow a_2 = \frac{-8}{144} = -\frac{1}{18}$

Equation #2 : $y = \frac{-1}{18}(x-24)^2+8$

b)

$y = \frac{-3}{32}(x^2-16x+64)+6$	$y = \frac{-1}{18}(x^2-48x+576)+8$
$y = -0.09375x^2+1.5x-6+6$	$y = -0.0556x^2+2.667x-32+8$
$y = -0.09375x^2+1.5x$	$y = -0.0556x^2+2.667x-24$

Solve these 2 quadratic equations.

Work is shown next page

$$
\begin{aligned}
& y = -0.0556x^2 + 2.667x - 24 \\
(-)\ & y = -0.09375x^2 + 1.5x \\
\hline
& 0 = 0.03815x^2 + 1.167x - 24
\end{aligned}
$$

$$x = \frac{-1.167 \pm \sqrt{(1.167)^2 - 4(0.03815)(-24)}}{2(0.03815)}$$

$$= \frac{-1.167 \pm 2.241}{0.0763}$$

$$\frac{-1.167 + 2.241}{0.0763} = \underline{14.08\,m}$$

$$\frac{-1.167 - 2.241}{0.0763} = \cancel{-ve}$$

reject the negative length!

$$y = \frac{-3}{32}(14.08 - 8)^2 + 6$$

$$= -3.468 + 6$$

$$= \underline{2.53\,m}.$$

Solution: (14.08 m, 2.53 m)

c) The solution represents the location and height of the support footing.

6. Caitlin is at the base of a hill with a constant slope. She kicks a ball as hard as she can up the hill.

a) Explain how the following system models this situation.

$$h = -0.09d^2 + 1.8d; \quad h = 0.5d$$

The 1st equation models the horizontal distance travelled and the height of the ball, it would follow a parabolic path that opens downward. The linear equation models the slope of the hill.

b) Solve the system.

$$h = -0.09d^2 + 1.8d$$
$$(-)\ h = 0.5d$$
$$0 = -0.09d^2 + 1.3d$$
$$0 = -0.09d(d - 14.4)$$

Solution: $d = 0$ or $d = 14.44$

$h = 0.5(0) = 0$ $\quad$ $h = 0.5(14.44) = 7.22$

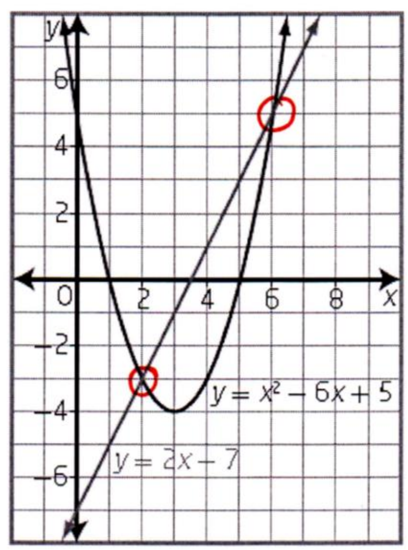
$(0, 0)$, $(14.44, 7.22)$

c) Interpret the point(s) of intersection in the context.

The point (0,0) represents the starting point, where the ball was kicked. The point (14.44, 7.22) represents the location where the ball landed on the hill.

7. a) Estimate the solutions to the system of equations shown in the graph.

Solutions: $(2, -3)$, $(6, 5)$



b) Solve the system algebraically.

$$y = x^2 - 6x + 5$$
$$(-)\ y = 2x - 7$$
$$0 = x^2 - 8x + 12$$
$$0 = (x-2)(x-6)$$

$x = 2$ or $x = 6$

$y = 2(2) - 7 = -3$ $\quad$ $y = 2(6) - 7 = 5$

$\boxed{(2, -3) \quad (6, 5)}$

8. Without solving the system $4m^2 - 3n = -2$ and $m^2 + \frac{7}{2}m + 5n = 7$, determine which solution is correct: $(\frac{1}{2}, 1)$ or $(\frac{1}{2}, -1)$.

*substitution

$4(\frac{1}{2})^2 - 3(1) = -2$

$1 - 3 = -2$ ✓

$(\frac{1}{2})^2 + \frac{7}{2}(\frac{1}{2}) + 5(1) = 7$

$\frac{1}{4} + \frac{7}{4} + 5 = 7$ ✓

$4(\frac{1}{2})^2 - 3(-1) \neq -2$

$1 + 3 \neq -2$

$(\frac{1}{2})^2 + \frac{7}{2}(\frac{1}{2}) + 5(-1) \neq 7$

$\frac{1}{4} + \frac{7}{4} - 5 \neq 7$

$(\frac{1}{2}, 1)$ is the solution!

9. Solve each system algebraically, giving exact answers.

a) $p = 3k + 1$

(−) $p = 6k^2 + 10k - 4$

$0 = 6k^2 + 7k - 5$

$0 = (2k - 1)(3k + 5)$

$k = \frac{1}{2}$ or $k = \frac{-5}{3}$

$p = \frac{5}{2}$ $\quad p = -4$

$(\frac{1}{2}, \frac{5}{2})$; $(\frac{-5}{3}, -4)$

b) $4x^2 + 3y = 1 \rightarrow (3y = -4x^2 + 1) \cdot 2$

$3x^2 + 2y = 4 \rightarrow (2y = -3x^2 + 4) \cdot 3$

$6y = -8x^2 + 2$

(−) $6y = -9x^2 + 12$

$0 = x^2 - 10$

$x = \sqrt{10}$ or $x = -\sqrt{10}$

$y = -13$ $\quad y = -13$

$(\sqrt{10}, -13)$; $(-\sqrt{10}, -13)$

c) $4 \cdot \left(\frac{w^2}{2} + \frac{w}{4} - \frac{z}{2} = 3\right) \rightarrow 2w^2 + w - 2z = 12$

$12 \cdot \left(\frac{w^2}{3} - \frac{3w}{4} + \frac{z}{6} + \frac{1}{3} = 0\right) \rightarrow 4w^2 - 9w + 2z + 4 = 0$

$2z = 2w^2 + w - 12$

(−) $2z = -4w^2 + 9w - 4$

$0 = 6w^2 - 8w - 8$

$0 = 2(3w^2 - 4w - 4)$

$0 = 2(3w + 2)(w - 2)$

$w = \frac{-2}{3}$ or $w = 2$

$z = \frac{-53}{9}$ $\quad z = -1$

$(-\frac{2}{3}, \frac{-53}{9})$ $\quad (2, -1)$

d) $2y - 1 = x^2 - x \rightarrow 2y = x^2 - x + 1$

$x^2 + 2x + y - 3 = 0 \rightarrow (y = -x^2 - 2x + 3) \cdot 2$

$2y = -2x^2 - 4x + 6$

(−) $2y = x^2 - x + 1$

$0 = -3x^2 - 3x + 5$

$x = \frac{3 \pm \sqrt{9 + 60}}{-6}$

$x = \frac{3 + \sqrt{69}}{-6}$ $\quad x = \frac{3 - \sqrt{69}}{-6}$

$y = \frac{-7 + \sqrt{69}}{6} + 3$ $\quad y = \frac{-7 - \sqrt{69}}{6} + 3$

See next page for determining $y = ?$

Substitute

$$x = \frac{3+\sqrt{69}}{-6}$$

$$\frac{(3+\sqrt{69})}{-6} \cdot \frac{(3+\sqrt{69})}{-6} = \frac{9+6\sqrt{69}+69}{36}$$

$$= \frac{78+6\sqrt{69}}{36}$$

$$y = -x^2 - 2x + 3$$

$$= -\left(\frac{3+\sqrt{69}}{-6}\right)^2 - \cancel{2}\left(\frac{3+\sqrt{69}}{\cancel{-6}\,3}\right) + 3$$

$$= \frac{-78-6\sqrt{69}}{36} + \frac{(3+\sqrt{69})^{\cdot 12}}{3 \cdot 12} + 3$$

$$= \frac{-78-6\sqrt{69}+36+12\sqrt{69}}{36} + 3$$

$$y = \frac{-42+6\sqrt{69}}{36} + 3 = \frac{-7+\sqrt{69}}{6} + 3$$

Substitute

$$x = \frac{3-\sqrt{69}}{-6}$$

$$\frac{(3-\sqrt{69})}{-6} \cdot \frac{(3-\sqrt{69})}{-6} = \frac{9-6\sqrt{69}+69}{36}$$

$$= \frac{78-6\sqrt{69}}{36}$$

$$y = -x^2 - 2x + 3$$

$$= -\left(\frac{3-\sqrt{69}}{-6}\right)^2 - \cancel{2}\left(\frac{3-\sqrt{69}}{\cancel{-6}\,3}\right) + 3$$

$$= \frac{-78+6\sqrt{69}}{36} + \frac{(3-\sqrt{69})^{\cdot 12}}{3 \cdot 12} + 3$$

$$= \frac{-78+6\sqrt{69}+36-12\sqrt{69}}{36} + 3$$

$$y = \frac{-42-6\sqrt{69}}{36} + 3 = \frac{-7-\sqrt{69}}{6} + 3$$

10. Manitoba has many biopharmaceutical companies. Suppose scientists at one of these companies grow two different cell cultures in an identical nutrient-rich medium. The rate of increase, S, in square millimeters per hour, of the surface area of each culture after t hours is modeled by the following quadratic functions:

First culture: $S(t) = -0.007t^2 + 0.05t$

Second culture: $S(t) = -0.0085t^2 + 0.06t$

a) What information would the scientists gain by solving the system of related equations?

By solving the system of related equations, the scientists would find the time when both cultures have the same rate of increase of surface area.

b) Solve the system algebraically.

$$y = -0.007t^2 + 0.05t$$
$$(-)\quad y = -0.0085t^2 + 0.06t$$
$$0 = 0.0015t^2 - 0.01t$$
$$0 = 0.0015t(t - 6.667)$$

solutions: $t = 0$ or $t = 6.667$

$S(t) = 0$; $S(t) = 0.022$

Solutions: $(0, 0)$, $(6.67, 0.022)$

At the time of 6.67 hours, both cultures have the same rate of 0.022 mm^2/h.